Date .....

Sequence & Series

2 4 6 8 10 → Sequence.
1 3 5 7 9 → Sequence.
2 3 5 7 11 → Sequence.
5 8 11 20 45 → No sequence.
49 25 5 1 → No sequence.

Eg:-

n=1	n=2	n=3	n=4	n=8 ?
1	3	5	7	___

$\boxed{2n-1} \Rightarrow 2\times8-1 = 15.$

2 3 5 7 11 ___

2 4 8 16 32 $\Rightarrow \boxed{2^n}$

Progression

A.P

Arithmetic ⇒ gap is same

1, 3, 5, 7 ....
2 2 2

$a_2 - a_1 = a_3 - a_2$

G.P

Geometric ⇐ Ratio same

3, 9, 27, 81 ....
3 3 3

$\frac{a_n}{a_{n-1}} = \frac{a_{n-1}}{a_{n-2}}$

Arithmetic Progression

Type-1 Based on General formula :-

eg :- 1 4 7 10. 13 ____ $n=10$?

$a_n \Rightarrow \boxed{a+(n-1)d}$ $\Rightarrow 1+(10-1)3 = 28$

$a=1$ $d=3$ $n=10$ $a_{10}=28$

$\Rightarrow a=1$ $a_n=28$ $n=10$ $d=?$

By BC

$28 = 1+(10-1)d \Rightarrow 3$

$\Rightarrow a_n=28$ $n=10$ $d=3$ $a=?$

By BC

$28 = a+(10-1)3 \Rightarrow 1$

$\Rightarrow a_n=28$ $d=3$ $a=1$ $n=?$

By BC

$28 = 1+(n-1)3 \Rightarrow 10$.

eg :- 1 4 7 10 $n=10$?

solu :- $a+d$ =

Type 2 General A.P

$a_n = a+(n-1)d$

$a_1, a+d, a+2d, a+3d$.....

eg :- $a_2=4$ $a_4=10$ $a_{15}=?$ $\rightarrow a+14d$

$a+d=4$ $a+3d=10$ General A.P=?

Date .....

$\Rightarrow a + d = 4$ $\quad a+3=4$

$a + 3d = 10$ $\quad a=1$

$-2d = -6$

$d = 3$

$\Rightarrow 1 + 14 \times 3 = 43$ Ans

General A.P:- 1, 4, 7, 10 .....

↳ First on calci. Find out....

Type 3

Sum given

- 3 terms → $a-d, a, a+d$.
- 4 terms → $a-3d, a-d, a+d, a+3d$.
- 5 terms → $a-2d, a-d, a, a+d, a+2d$.

→ odd terms → gap d.

→ even terms → gap $2d$.

eg:- Sum = 69 (of Three Terms) Product of first two Terms 483

$\Rightarrow a-d + a + a+d = 69$

$3a = 69$

$a = 23$

$\Rightarrow (a-d)(a) = 483$

$(23-d)23 = 483$

$23-d = 21$

$23-21 = d$

$d = 2$

$\Rightarrow 23-2, 23, 23+2$

$\Rightarrow 21, 23, 25$

Date .....

Type 3 :- B.C :-
↳ Form A.P.

obs (21, 23), 25 ⇒ form A.P
↓ ↳ 68 ✓
✓ 483

Type 4 — A.M (i) single A.M.

eg :- 2 (A) 6 (A.P)

$A - 2 = 6 - A$

$2A = 2 + 6$

$A = \frac{2+6}{2} = 4$

$\boxed{\frac{a+b}{2}}$

(ii) more than one A.M inserted b/w a & b :-

eg :- 4 A1 A2 A3 A4 19 $\quad n = 6$

$a_6 = 19$

$a = 4$

$\Rightarrow a_6 = a + 5d$

$19 = 4 + 5 \times d$

$19 - 4 = 15 = 5d$

$\frac{15}{5} = d \Rightarrow 3$

$A1 = a2 = a + d = 4 + 7 = 11$

$A2 = a3 = a + 2d$ $\Rightarrow \boxed{An = a + nd}$

$A3 = a4 = a + 3d$

$A4 = a5 = a + 4d$

7 10 13 16

⇒ 4 A1 A2 A3 A4 19

A.M (d find out)

$d = \frac{b-a}{n+1} = \frac{19-4}{5} \Rightarrow d = 3$

b = last term.

a = first term.

n = No of A.M's inserted b/w a & b

Spiral

Date .....

calci.. by → 4 A_1 A_2 A_3 A_4 19
If B.C
↳ 4+3 = = = = =

Type 5

eg:- 1 4 7 10 _ _ _ _

$n = 10$ sum ?

$$\Rightarrow \boxed{S_n = \frac{n}{2}[2a+(n-1)d]} \quad a=1 \quad d=3 \quad n=10$$

$$= \frac{10}{2}[2+(9)3]$$

$$\Rightarrow 145 \text{ Ans}$$

calci :- a+d = = = = = = =
= = = GT + a
Button

$\Rightarrow$ a+d = = = ... 9T + a
2nd Term

eg:- $S_{10} = 145$ $a=1$ $d=3$ $n=10$
(i) $S_{10} = 145$ $d=3$ $n=10$ $a=?$
(iii) $S_{10} = 145$ $d=3$ $a=1$ $n=?$
(iii) $S_{10} = 145$ $a=1$ $n=10$ $d=?$

(i) $S_n = \frac{n}{2}[2a+(n-1)d] \Rightarrow 145 = \frac{10}{5\cancel{2}}[2a+9\times3]$

$$\Rightarrow \frac{145}{5} - 27 = 2a$$

$$\Rightarrow 29-27 = 2a$$

$$\cancel{2} = \cancel{2}a$$

$$a = 1$$

Date .....

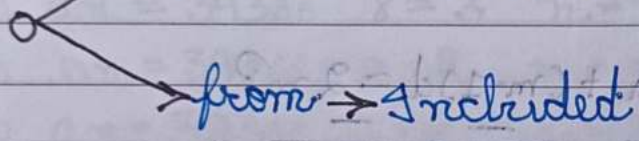
(ii) $S_n = \frac{n}{2}[2a+(n-1)d]$

If Bc $145 = \frac{n}{2}[2\times1+(n-1)3]$

$290 = 2n + 3n^2 - 3n$

$3n^2 - n - 290 = 0$

Type 6 Find out n Sum Divisible by ....



$S_n = \frac{n}{2}[a+a_n]$

⇒ If last term given.

eg:- Sum 100 ..... 1000 Divisible by 5 between.

Between → 105, 110 ---- 995

from → 100, 105 ---- 1000

eg:- 100 .... 1000 Divisible by 5

Between → 105, 110 ------ 995

$n = 179$

$\Rightarrow S_n = \frac{n}{2}[a+a_n]$

$= \frac{179}{2}[105+995] = 98450$

$n = 995 = 105 + (n-1)5$

$= \frac{995-105}{5} + 1 = 179$

from → 99550

Date .....

eg:- 1000 ------ 10,000 Divisible by 13

BTW From → 1001 ------ 9997

$\frac{1000}{13} = 76.9$ + 1 ⇒ 77×13

$\frac{10,000}{13} = 769.2$ ↓ 769×13

eg:-

$S_n = n^2P$ $S_m = m^2P$ S_P

$= \frac{n}{2}[2a+(n-1)d] = n^2P$ $= 2a+(n-1)d = 2nP$

$= \frac{m}{2}[2a+(m-1)d] = m^2P$ $= 2a+(m-1)d = 2mP$

$d(n-1-m+1) = 2P(n-m)$

$d = 2P$

$\Rightarrow 2a+(n-1)2P = 2nP$

$2a+2nP-2P = 2nP$

$2a = 2P$

$a = P$ $S_P = \frac{P}{2}[2P+(P-1)2P]$

$= P[P+P^2-P]$

$\Rightarrow P^3$

Geometric Progression [G.P. series]

Type-1

Common Ratio

eg:- 1 3 9 27 81

$a = 1$ $\frac{3}{1} = 3$ $\frac{9}{3} = 3$

$a_{10} = ?$

$r = 3$

$\Rightarrow$

$$\boxed{a_n = a \times r^{n-1}}$$

Date .....

eg:- $a = 4$ $r = 1/2$ $a_{10} = ?$

$a_n = 4 + 1/2^9 = \frac{4}{2^9} = \frac{4}{512\ 128} \Rightarrow \frac{1}{128}$ Ans

eg:- $a = 2$ $r = 3$ $a_{10} = ?$

$a_n = 2(3)^9 = 39366$

$$\boxed{a_n = a r^{n-1}}$$

eg:- $a_n = 39366$ $a = 2$ $r = 3$ $n = 10$

(i) $a_n = 39366$ $r = 3$ $n = 10$ $a = ?$

(ii) $a_n = 39366$ $a = 2$ $n = 10$ $r = ?$

(iii) $a_n = 39366$ $a = 2$ $r = 3$ $n = ?$

(i) $a_n = ar^{n-1} = 39366 = a(3)^9 \Rightarrow 2$ Ans

(ii) $a_n = ar^{n-1} = 39366 = 2(r)^9 = 19683 = r^9$ by P.C $\Rightarrow 3$ Ans

$\Rightarrow 19683 \sqrt{}$ 12 times $-1 \div 9 + 1 = (r \Rightarrow$ 12 times

(iii) $a_n = ar^{n-1} = 39366 = 2(3)^{n-1} = 19683 = 3^{n-1} \Rightarrow n-1 = 9$

$3^9 \Rightarrow n = 10$

Type 2:-

General G.P $a_n = ar^{n-1}$

$a, ar, ar^2, ar^3 \ldots\ldots$

eg:- $a_2 = 3$ $a_4 = 27$ $a_8 = ? \; ar^7$

$ar = 3$ — ① $\quad ar^2 = 27$ — ② General G.P.

② $\div$ ① $\quad \frac{r^2 ar^3}{ar} = \frac{27}{3}\ 9 \quad r^2 = 9$

$r = \pm 3$

$r = 3 / a = 1 \quad a_8 = 1(3)^7 = 2187$

$r = -3 / a = -1 \quad a_8 = -1(-3)^7 = 2187$

Spiral

Date .....

Type 3:- Product of terms

3 terms → a/r, a, ar,
4 terms → a/r^3, a/r, ar, ar^3.
5 terms → a/r^2, a/r, a, ar, ar^2.

eg:- $\boxed{a_n = a r^{n-1}}$

$a_n = 2(3)^6$
$= 1458$

$a = 2$ $r = 3$ $n = 7$ $a_n = ?$

calci → (r) ^ (a) = = = = = =
2nd term

Find $a = ?$ → ly B.C / calci.
Find $r = ?$ → ly B.C / calci.
Find $n = ?$ → ly. B.C / calci / Rinert.

Type 4 :- i) 2 & 8 ly. P

$\Rightarrow \frac{ly}{2} = \frac{8}{ly}$ $ly^2 = 2 \times 8$
$ly = \sqrt{2 \times 8}$

$\Rightarrow$ a ly b
$\frac{ly}{a} = \frac{b}{ly}$ $ly^2 = ab$
$ly = \sqrt{ab}$

ii) More than one ly. n inserted :-

$\Rightarrow$ 2 ly1 ly2 ly3 ly4 64
$a = 2$

ly1 = a2 = ar ly3 = a4 = ar^3
ly2 = a3 = ar^2 ly4 = a5 = ar^4 $\Rightarrow$ $\boxed{G_n = a r^n}$

Spiral

Date .....

$a_6 = 64$

$2r^5 = 64 \Rightarrow r^2 = 32 \Rightarrow r = 2$

Shortcut:- more than one Ly. m.

a Ly1 Ly2 ... Lyn b

Ly. m → to find out

$$r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$$

n = odd = ± Both sign

b = last term n = No of Ly. m's

Type-5:- S_n

$$a + ar + ar^2 + \text{.....}$$

- $S_n = \frac{a(r^n - 1)}{r - 1} \longrightarrow r > 1$
- $S_n = \frac{a(1 - r^n)}{1 - r} \longrightarrow r < 1$
- $S_n = n \times a \longrightarrow r = 1$

eg:- 2 4 8 16 ..... $S_{15} = ?$

$a = 2 \quad r = 2 > 1 \quad n = 15$

$$\Rightarrow S_{15} = \frac{a(r^n - 1)}{r - 1} = \frac{2(2^{15} - 1)}{2 - 1} = 65534.$$

eg:- 2 6 18 54 ..... $S_{10} = ?$

$a = 2 \quad r = 3 \quad n = 10$

$$S_{10} = \frac{2(3^{10} - 1)}{3 - 1} = 59048.$$

Spiral

Date .....

(ii) $n=10$ $S_n=59048$ $a=2$ $r=?$

(iii) $S_n=59048$ $a=2$ $r=3$ $n=?$

(iiii) $S_n=59048$ $n=10$ $r=3$ $a=?$

(ii) $59048 = \frac{2(3^n-1)}{3-1} \Rightarrow 59049 = 3^n$

$n=10$

(iiii) $59048 = \frac{a(3^{10}-1)}{3-1} \Rightarrow a=2$

(ii) $59048 = \frac{2(r^{10}-1)}{r-1}$ by B.C $\Rightarrow$ Put Value of $r=3$

Type 6:- Infinite G.P Sum

1mk

$1 + 1/2 + 1/4 + \dots\dots\dots$

$S_\infty = \frac{a}{1-r} \rightarrow r<1$

$a=1$ $r=\frac{1}{2}$ $\frac{1}{1-1/2}=2$

* $\boxed{S_n = Pn^2+Qn}$ A.P

$\Rightarrow S_n = 3n^2+2n$

$a=5$ $d=6$

$\frac{n}{2}[10+(n-1)6]$ $\leftarrow$ [5, 11, 17....]

$S_1 = 3+2=5$

$(3n+2)n$ $n[5+3n-3]$ $S_2 = 16$

$S_2 - S_1 = a_2 = 16-5=11$

Date .....

* $S_n = Pn^2 + Qn$ A.P

$\Rightarrow a = P+Q \quad d = 2P$

* $\boxed{a_n = Pn+Q}$ linear A.P

$\Rightarrow a = P+Q \quad d = P$

* $a_m = Pm+Q$.

$\Rightarrow A.M \geq G.M \geq H.M$

$\Rightarrow G.M^2 = A.M \times H.M$

A.M G.M H.M

G.P

a A b a G b a H b

$A = \frac{a+b}{2}$ $G = \sqrt{ab}$ $H = \frac{2ab}{a+b}$

eg 2 8

$A = 5$ $G = 4$ $H = 3.2$

$5, 4, 3.2$ = G.P

$G^2 = A.H$

Shortcut:- $\frac{1}{a_1 \cdot a_2} + \frac{1}{a_2 \cdot a_3} + \frac{1}{a_3 \cdot a_4} + \text{---------}$

$a_1 a_2$ ------ A.P

$S_n = \frac{n}{a_1(a_n+1)} = \frac{n}{a(a+nd)}$

RITU JINDAL

MIND MAP

SEQUENCE AND SERIES

Arithmetic Progression
⇓
Common difference

Calci
$a_n = a + (n-1)d$

- $a_2 = ?$ $a_4 = ?$ $a_n =$ or general A.P. a and d Calci
- Sum of 3 terms, 4 terms, 5 terms: $a-d, a, a+d$ or GBC
- A.M
  - Single → $\frac{a+b}{2}$
  - More than One → $d = \frac{b-a}{n+1}$, n = No of A.M's or Calci

Calci
$S_n = \frac{n}{2}[2a+(n-1)d]$ or $S_n = \frac{n}{2}[a+a_n]$

⇒ Sum of n A.M's = n · Single A.M. = $n \cdot \frac{(a+b)}{2}$

Geometric Progression
⇓
Common Ratio

Calci
$a_n = a r^{n-1}$

- $a_2 = ?$ $a_4 = ?$ $a_n =$ or General G.P. a on r. GBC Calci
- Product of 3 terms, 4 terms: $\frac{a}{r}, a, ar$ or GBC
- G.M
  - Single → $\sqrt{ab}$
  - More than One → $r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$, n = No of G.M's or Calci.

Calci
$S_n = \frac{a(r^n-1)}{r-1}$
⇓
$S_\infty = \frac{a}{1-r}$ $r<1$

⇒ Product of n G.M's = n (Single GM) = $(ab)^{n/2}$

Some Important Formulae

$1+2+3+\cdots+n = \frac{n(n+1)}{2}$

$1^2+2^2+3^2+\cdots+n^2 = \frac{n(n+1)(2n+1)}{6}$

$1^3+2^3+3^3+\cdots+n^3 = \left(\frac{n(n+1)}{2}\right)^2$

$1+3+5+\cdots+ = n^2$

$2+4+6+\cdots+ = n(n+1)$

- $S_n = Pn^2 + Qn$ ⇓ $a = P+Q$, $d = 2P$
- $a_n = Pn + Q$ ⇓ $a = P+Q$, $d = P$